

# OPTIMIZATION WITH PENNES' BIOHEAT EQUATION AS CONSTRAINT

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In this paper, an energy optimization problem with Pennes' bioheat equation as a constraint is solved numerically. The solution of optimal control problem for the Pennes' bioheat equation involving boundary function of continuous time variable in its energy function is proposed. A specific direct approach based on Control parameterization method is suggested to solve optimal boundary control for Pennes' bioheat equation. Using space discretization one obtains an ODE which is then solved by using fourth-order Runge-Kutta formula. Now we deal with a linear system of algebraic equations based on  $b_i$ ,  $i = 1, \dots, N$ , the unknown parameters of control function, in which they are considered as  $N$  constraints in the optimization problem. This optimization problem has nonlinear objective function with linear constraints that is solved by Rosen's gradient projection technique. Calculation of the optimal values for parameters yields to the optimal control for the unknown boundary function. Numerical results for this process of optimization with PDE (Pennes' Eq.) constraints are presented.

## References

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