MATHEMATICAL ABILITIES: IDENTIFICATION AND DEVELOPMENT

Mihajlović A., Egerić M., Dejić M.

Successful differentiation and individualization of teaching mathematics depends greatly on teacher's knowledge of mathematical abilities, their components and classification. The paper aims to present brief theoretical basis of mathematical abilities, to verify some of them in the results of experiments, and to suggest the modes of developing mathematical abilities.

The first part of the paper, discusses different definitions, nature and structure of mathematical abilities. It then gives a brief overview of similarities and differences in the main conceptions about mathematical abilities. The second part of the paper presents results of the research that intends to explore the relationship between the development of mathematical abilities in primary school students (10 – 11 years old) and their gender, intelligence, grades in mathematics, results on mathematics knowledge test, educational status of their parents. Comparison between nominations of parents, primary teachers and peers is made. The final part of the paper, discusses different ways of developing mathematical abilities in the classroom. It proposes the content that can be used for developing and fostering mathematical abilities in primary school children.

Introduction. Mathematical abilities are studied by psychologists, pedagogues, mathematicians, methodologists and others, from various aspects. Mostly, the subjects they are dealing with refer to studying of thinking processes manifested at solving different mathematical problems and existence of general (group) or specific mathematical abilities. Generally, the researches into mathematical abilities can be classified in two groups: *introspective* and *factorial.* Singling out mathematical abilities' components (parameters) is set as a final aim of the researches. The purpose of our work was to define briefly mathematical abilities, explore experimentally the connection between the level of mathematical abilities development and gender of the students, their general intellectual capacities, success in mathematics and educational status of their parents, and point to the possibilities of developing mathematical abilities in children. Presentation of the research results is a target subject, and other issues will be worked out to the extent necessary for better understanding of these results. In our research, when mathematical abilities are in question, we bear in mind, above all, the pupils who can be classified into a category of gifted ones, thus, the ones with high mathematical abilities.

Theoretical Background. One of the initiators of researching mathematical abilities was Henri Poincare, (1854-1912), a prominent French mathematician. He found out the existence of special mathematical abilities and pointed out to their important component – mathematical intuition. Depending on the level of possessing mathematical intuition, Poincare singled out three levels of mathematical abilities.

Since the beginning of the 20th century till present days, scientists all over the world have made various theories of mathematical abilities, setting aside the components which are

integral parts of these abilities as a final result.The research works have beeen made in several directions:

1. The structure of mathematical abilities has been explored,
2. A theoretical- psychological interpretation has been given,
3. Parameters of specific mathematical abilities have been determined,
4. Mathematical abilities have been searched for in general.

Some research works present subjective opinions of famous people, for others, components of mathematical abilities are insufficiently studied, both separately and on the whole, but there are also quite serious and complete researches, and they are scarce. On the one hand, there are research works that use test and factorial analysis, and on the other introspective researches. By using only one of these research methodologies, there arise insufficient solutions of a stated problem. Factorial analysis does not give clear and substantial enough image of mathematical abilities structure. Introspective researches have a certain amount of subjectivity and superficiality within themselves. Philosophical and psychological orientations of a research worker determine certain components of mathematical abilities here. Among the researchers themselves there are disputes about participation of certain components which make mathematical abilities, and their definitions are not in correlation. All the above disables giving a complete definition and unique answer to the question what mathematical abilities are.

Metelskii(1989), through studying researches of mathematical abilities presented at the conference in the USA in 1952, out of 11 different opinions, separated the following factors of mathematical abilities structure (number of opinions out of 11 observed is given in parentheses):

1. The power of abstracting, operating abstractions (10);
2. Spatial factor (6);
3. Mathematical intuition, anticipation of results (5);
4. Flexibility of opinion (5);
5. Clear logical opinion (3);
6. Numerical factor (3).

Further on, Metelskii(1989) briefly observed 20 independent conceptions of mathematical abilities more, which emphasized 30 different components of mathematical abilities, among which the most often mentioned are:

1. The power of abstracting and operating abstractions (13)
2. Spatial factor or geometrical intuition (9)
3. Logical reasoning (8)
4. Flexibility, inventive mind (7)
5. Mathematical intuition (6)
6. Numerical factor (4)
7. Analysis and synthesis (3)
8. Aspiration towards rationality in solving assignments (3)
9. Generalization, finding similarities in different (3)

Combinatory, deductive and inductive opinion, accuracy in symbolics, knowing how to apply mathematical schematism, inclination to and interest in mathematics, adoption of

mathematical ideas, etc. are also included in the components important for adoption of mathematical knowledge.

Dejić, M. and Bandjur, V. (2006) have found, among the 28 elaborated conceptions where more than 30 parameters are mentioned, the following ones as the most often quoted:

1. Spatial factor (8)
2. Logical reasoning (8)
3. Abstracting , operating abstractions (7)
4. Generalization (6)
5. Ability of using symbols (6)
6. Memorizing (5)
7. Deduction (5)
8. Analysis, synthesis (5)
9. Resourcefulness (4)
10. Numerical factor (4)
11. Process of thinking quickness (3)
12. Critical opinion (3)

The shown components of mathematical abilities, mutually often different, give together a complete image of an observed problem. They can be grouped in various ways:

1. According to mathematical fields,
2. According to mathematics as unique science,
3. According to general abilities,
4. Schooling and scientific abilities,
5. Abilities to solve mathematical assignments,
6. Abilities to create mathematical ideas,
7. Abilities to find out, etc.

Complete mathematical abilities components *classification* is given by Gusev, a Russian mathematician and pedagogue (2003). He himself has not given any conception of mathematical abilities, but, on the basis of those presented in his book, he has made a classification of parameters and it is one of the most complete given so far of which we know.

Identification of highly developed mathematical abilities in schoolchildren. Another problem concerning mathematical abilities is how to identify pupils with highly developed mathematical abilities. How to recognize mathematically gifted pupils? An important role in this process is given to the opinion and nominations of teachers, but also of parents and peers. Still, both teachers and parents can hesitate as to what characteristic they should pay attention. Some researches indicate that teachers can not recognize all gifted students, and sometimes they proclaim some non-gifted as gifted. Moreover, we must be cautious when taking results of school tests into consideration. Not all children that have high performance at school tests are necessarily mathematically gifted. School tests are usually directed towards checking calculation skills and not mathematical thinking abilities. Very often IQ scores are taken as one of predictors and identificators of giftedness. However, they are not so reliable, because the results are related to a variety of different abilities and skills, and only some of them are connected with mathematical abilities and giftedness. One more way to identify mathematically

gifted students are results on creativity and achievement tests. Though, achievement tests usually test calculation skills and should, therefore, be regarded with caution.

In our research we have tried to examine the concord of some common ways of identification of students with highly developed mathematical abilities.

Research methodology. The main goal of our research was to explore the relationship between the development of mathematical abilities in primary school students (10 – 11 years old) and their gender, intelligence, grades in mathematics, results on mathematics knowledge test, educational status of their parents. We have also tried to explore if there are differences between boys and girls in the level of development of some components of mathematical abilities. The research has been conducted in five primary schools in Jagodina. Testing of 150 fourth grade students has been done with the following tests: intelligence test (Raven's Progressive Matrices) and knowledge test. Questionnaire survey has included all students, their parents and teachers. Opinions and nominations about students with highly developed mathematical abilities, respectively mathematically gifted students are given by teachers, parents and peers.

Knowledge test consisted of questions and tasks related to the mathematics syllabus of the fourth grade of primary school.

Mathematical abilities test consisted of 12 tasks, which were divided into four categories due to the mathematical abilities components that dominated in the process of their solving: 1. Tasks that demanded developed arithmetical-numerical component; 2. Tasks that demanded ability of making abstraction and generalization; 3. Tasks that demanded developed geometrical component and 4. Tasks that demanded developed logical-combinatorial component.

Based on results of testing (intelligence test, knowledge test), grades in mathematics, nominations of teachers, parents and peers, sub-sample of 60 potentially mathematically gifted students has been selected.

Gender	Number of students	%
Male	34	56,7%
Female	26	43,3%
Total	60	100%

After testing 60 students of sub-sample with mathematical abilities test, 40 students have been marked as students with highly developed abilities, respectively as mathematically gifted.

Hypotheses of our research were:

1. It is supposed that there is no full concord between nominations of teachers, parents and peers, and there is no complete accordance between these nominations and results on mathematical abilities test.
2. It is supposed that there is relationship between the highly developed mathematical abilities in students and their intelligence.
3. It is supposed that there is relationship between the highly developed mathematical abilities in students and their results on mathematics knowledge test.

4. It is supposed that there is relationship between the highly developed mathematical abilities in students and their grades in mathematics.
5. It is supposed that there is no relationship between the highly developed mathematical abilities in students and their gender.
6. It is supposed that there is no relationship between the highly developed mathematical abilities in students and educational status of their parents.
7. It is supposed that there are typical characteristics of primary school students with highly developed mathematical abilities

The results of our research are the following:

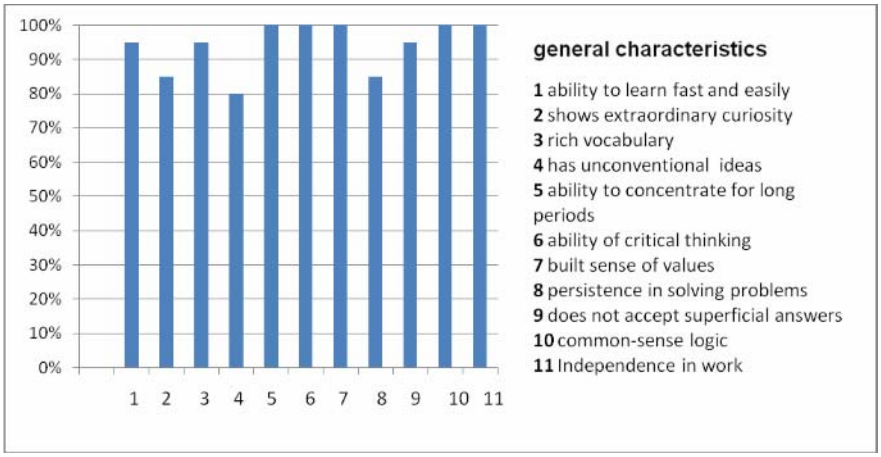
1. One of the criteria for identifying students with highly developed mathematical abilities were nominations of teachers, parents and peers. Based on comparative analyses of these nominations, it appeared that teachers have missed to identify a half of the students with highly developed mathematical abilities. They have identified 50% of students that had high performance on mathematical abilities test. Peers nomination were slightly better, they have identified 57,5% students with high performance in mathematical abilities test. It appears that peers know each other better than teachers do. The best in nominations were parents, who identified 62,5% of students with high performance at mathematical abilities test. It was expected that parents would be on the less successful in identification due to their subjectiveness.
   By comparing these nominations, we have find out that there is accordance among them in only 25%.
2. As one more factor that influence on manifestation of mathematical abilities appeared to be intellectual abilities of students. Therefore, between performance on mathematical abilities test and intellectual abilities of students there is a positive relationship ($r_{\cos pi} = 0{,}4965$). However, from the frequencies we see that among students with average intellectual abilities there are some that have high performance on mathematical abilities test, and vice versa. Not all students with high intellectual abilities performed well in mathematical abilities test. Thus, we cannot accept intelligence test as an absolute predictor of mathematical abilities development.
   Some other authors (B. Djordjevic, 1999) share a similar opinion. They think that intellectual giftedness is one of criteria of giftedness, but not sufficient. In order to make identification of gifted better it is necessary to use some special abilities tests too. Great number of psychologists consider that at a younger age broader criteria of giftedness should be used. I.Ivic (B. Djordjevic, 1999) has found out in his research that there are differences in living condition of children in Serbia, which can leave deep influence on development of intelligence that are examined with traditional intelligence tests.
3. Based on results of our research we have found out that there is no relationship between performance on mathematics knowledge test and performance in mathematical abilities test ($C = 0{,}21$, $df = 2$, $\chi^2 = 2{,}86$). Some students who achieved excellent results in mathematics knowledge test, performed low scores on mathematical abilities test. Also, some students that performed low scores at mathematics knowledge test, had high performance at mathematical abilities test. Therefore, we can conclude that

knowledge tests cannot be used as reliable indicator of mathematical abilities development.

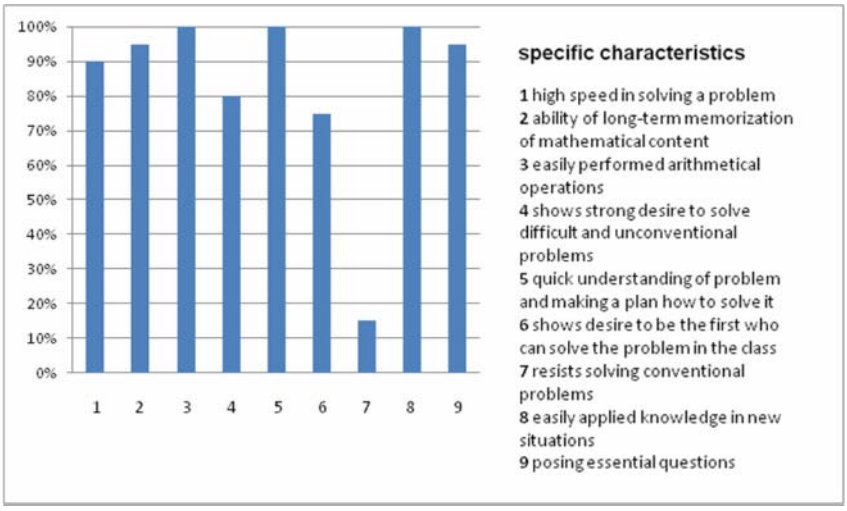
4. Our hypothesis about relationship between grades in mathematics and development of mathematical abilities has shown to be correct. We have found statistically significant difference ($r_{\cos pi} = 0{,}37$). Students with high grades had better performance at mathematical abilities test. Nevertheless, we must take into consideration that among students with lower grades in mathematics are also students with high performance at mathematical abilities test. Similarly, not all students with high grades have high performance at mathematical abilities test. High grades in mathematics cannot be the proof of highly developed mathematical abilities.
5. Results has shown that student gender does not influence the level of mathematical abilities development. Boys and girls have achieved similar results on mathematical abilities test, and we have not found statistically significant difference ($\phi = -0{,}12$, $df = 1$, $\chi^2 = 0{,}85$). B. Djordjevic (1995) obtained similar results in exploring problem of failure of gifted students. She determined that there is no statistically significant difference between gender and manifestation of potential giftedness. Patricia Campbell (1995) says that differences between individual girls or between individual boys are greater than those between average girl and average boy.
   We encountered similar results when exploring the relationship between development of particular mathematical abilities components (arithmetical-numerical**,** abstraction and generalization component, geometrical and logical-combinatorial component) and gender. We have not found statistically significant difference as well. Some authors consider that boys are better at tasks that demand three-dimensional presentation and spatial ability (Horne, 2004). Royer (Geary, 1999) and others, find out that male fourth graders were faster than female fourth graders in solving arithmetical and textual problems. Zech (1998), relying on Menacher's work (1994), referring to success in learning mathematics in relation to gender, points out that some differences exist, but research results are not uniform. Research results depend on a series of different factors, such as parents, type of task and sort of used test. Generally, boys have been found to be more successful. The greatest difference was found in solving tasks that demanded spatial abstraction and in solving arithmetical tasks. This was related with greater experience of boys in constructing and making different objects. Differences are obviously less when girls are given more time for solving tasks. This can be explained by the fact that girls approach solving problems with less self-confidence. However, we cannot assume that there are biological differences between boys and girls in mathematical giftedness only relying on these little differences. There are more and more opinions that differences in mathematical abilities of boys and girls are related to social and cultural aspect, and not to gender. Some PISA results have shown that there is no statistically significant difference in success in mathematics between boys and girls. Thus, in three countries, such as New Zealand, girls had better performance than boys.
6. Regarding our hypothesis that there is no relationship between development of mathematical abilities in students and educational status of parents, we have encountered interesting results. Educational status of father does not have influence on ap-

pearance of mathematical abilities, since we have not found statistically significant difference ($r_{\cos pi} = 0{,}04$). On the other hand, we have found that there is relationship between educational status of mothers with results on mathematical abilities test. Students whose mother have higher education, have higher performance at mathematical abilities test. We might explain this by the fact that educational status of women have become better nowadays. Also, mothers spend more time with their children than fathers do. We should mention that B. Djordjevic (1995) has shown that there is relationship between educational status of both parents and their children's success on abilities tests.

7. Based on results of our research we have found some characteristics typical for fourth grade students with developed mathematical abilities. According to teachers, general characteristics are:
   Independence in work (100%), built sense of values (100%), ability of critical thinking (100%), ability to concentrate for long periods (100%), and common-sense logic (100%). Majority of students show characteristics such as: ability to learn fast and easily (95%), and rich vocabulary (95%).



As for specific characteristics, teachers have encountered the following: easily performed arithmetical operations (100%), quick understanding of problem and making a plan how to solve it (100%) and easily applied knowledge in new situations (100%). A big number of students also show abilities such as: ability of long-term memorization of mathematical content (95%), posing essential questions (95%) and high speed in solving a problem (90%).



How to Develop Mathematical Abilities in Children. In order to answer the question how to develop mathematical abilities in children, first we have to give the answer if there are general mathematical abilities or just individual ones. Unfortunately, disputes among scientists occur here. Some of them deny, the others confirm the existence of mathematical abilities. It is our opinion that the existence of mathematical abilities should be noticed, and their development should be approached from that standpoint. Combinatory abilities are required in one field of mathematics, logical opinion in the second, abilities to find out the best algorithms for counting in the third, somewhere geometrical intuition is required, etc. Respecting separated components of mathematical abilities, we can say that they will develop in children if it is made possible for them to operate mathematical symbols, if they are released from concrete images quickly and pass to abstractness, if habits, persistence, patience in work are formed in them, if they state mathematical facts rationally, express themselves in a laconic way, generalize, etc.

When a teacher finds out potential mathematical abilities in children, he should continue revealing and working out certain components of these abilities, discover the most expressive ones and develop them. For that purpose, various forms of differentiated teaching which provide optimal realization of teaching individualism principle in class should be used.

Adapting curriculum to gifted pupils means differentiation at all levels (contents, process, results). One of differentiation aspects for gifted pupils is enriching and acceleration, where enriching represents horizontal expanding of standard curriculum, and acceleration stands for vertical rise. It is ideal when both approaches are appllied, as one supplements the other. It is also important that all activities are conducted continuously.

We can also influence development of mathematical abilities in children by solving unstandard and non-routine problems (J.Dunlap, 2001). A non-routine problem is the one which requires from the person who solves it to apply mathematical reasoning in order to find out algorithm different from that learned out in classes. A pupil is asked to create a method unique for the problem by using parts of similar problems and algorithms. Solving of the problems demands from the teacher not only to examine the solution, but also the method of finding the solution. These types of problems develop mathematical thinking and

mathematical abilities in children, because children have to synthetize information and make intuitive leaps, such as, for example, which method would be appropriate, and which would not.

Posing problems is a different, but not at the least less valid tool for developing mathematical thinking and abilities. Moses (J.Dunlap, 2001) recommends several ways how to accelerate development of creative mathematical thinking in children through posing problems. The first way is modifying of existing problems in the textbooks. Altering problems found in most of the textbooks, we can modify them into a greater challenge for pupils and demand from them to modify known algorithms in order to solve them. The second way recommended by Moses is using questions and tasks for which more answers are possible. Problems with one correct answer only and one obvious way of finding a solution do not encourage creative mathematical thinking and do not develop abilities , as children use already known algorithm.

It is also important that a problem should be beyond knowledge base of the person who solves it. Therefore, if a problem is not beyond a sphere of previously discovered facts, then it does not present a challenge for the person who solves it. If a „problem“ does not exist, a pupil does not gain any new knowlegde, does not find his own ways of finding solutions and there is no mathematical thinking development.

Conclusion. In order to make successful differentiation and individualization of teaching mathematic, besides knowing students' mathematical abilities and identifying them, teachers should also be familiar with strategies of fostering them. When identifying a student with highly developed mathematical abilities, several criteria should be applied. Opinion and nominations of teachers, parents and peers are important, but can be subjective. Although there is a positive relationship between development of mathematical abilities and intellectual abilities of students, we cannot accept intelligence test as absolute predictor of mathematical abilities development. Similarly, though there is relationship between grades in mathematics and development of mathematical abilities, we cannot regard as identical school and scientific mathematical abilities. Since differences between boys and girls in level of development of mathematical abilities are small, we may say that gender has no influence on existence of highly developed mathematical abilities. There is a relationship between highly developed mathematical abilities and educational status of mothers, which can be justified with the fact that mothers spend more time with their children than fathers. Based on results of research we can also conclude that there is no relationship between results on mathematical knowledge test and results on mathematical abilities test. This is a reason why we cannot use knowledge tests as indicators of highly developed mathematical abilities. We can influence development of mathematical abilities with solving unconventional tasks and with independent production of problems. Teachers can prepare different sets of tasks by themselves, which they can use to foster development of particular components of mathematical abilities.

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