THE ORIGIN OF FRACTALITY OF STOCK MARKET TIME SERIES

Krivosheev O.I.

ICP RAS, Profsoyuznaya 65, Moscow, RSXX0063, Russia, 8(926)14-777-36,
okrivosheev@mail.ru

We consider several strictly proved facts on smooth continuous functions presenting asset price. In order to prevent some mathematical difficulties we consider time functions instead of discrete time series. We show that no smooth function can present market prices at least if there is unlimited and cheap credit for riskless borrowers and no transaction costs.

We show that under these conditions are possible strategies that provide infinite return at low or even at zero risk. So smooth price function in continuous model is impossible as it violates the no-arbitrage condition.

We show that the value φ_r (more o less depending on $\vec{x}$ set if row dimension is not 0.5)

$$\varphi_r = \sum_i \left(\frac{\Delta x_i}{x_i}\right)^2 \text{ -} \qquad (1)$$

one can consider as a fractal resistivity of time series.
where x_i is a price value at some point & $x_i - x_{i-1}$ is it's increment Δx_i.

We prove a some kind of Ohm's low for market

$$\ell^* = 1/2 + \frac{\ln \frac{x_N}{x_0}}{\varphi_r} \qquad (2)$$

where ℓ^* is credit leverage – ratio of invested capital to own one,
$\frac{x_N}{x_0}$ - final to initial price ratio at the sample.

ℓ^* is proved to be (nearly) optimal leverage and can be calculated for expected values of φ_r and $\frac{x_N}{x_0}$.

We show that nearly smooth continuous time functions (with low fractal resistivity) are impossible too, if they provide too high return rate – more than equilibrium one for the market. Then we use these facts as a kind of heuristics to understand real market behavior especially to get ½ (or1 ½) price time series dimension.

References

1. *Кривошеев О.И.* Поиск оптимального кредитного рычага в условиях максимизации ожидаемой скорости роста стоимости портфеля. М.: Проблемы управления Вып.6 за 2015г. – с.35-45.*(Krivosheev O.I. Finding optimal financial leverage maximizing portfolio value growth rate- Moscow. Problems of control. 2015 Vol.6 – pp 35-45)*