ON THE ESTIMATION OF EXPONENTIAL SUMS DEALT WITH LATTICE POINTS DISTRIBUTION IN THREE-DIMENSIONAL AREAS

Arkhipova L.G.

The Moscow state university of M.V. Lomonosov, fac. of mathematics and mechanics, sec. of Mathematical analysis, Russia, 119992, Moscow, Lenin hills 1. phone: (495) 939-18-01

When we estimate error terms in the sphere problem and in problem of averages of class number of imaginary quadrate fields, we should be interested in estimation of exponential sums S like this

$$S = \sum_{v \le k,m,n, \le 2v} e^{2\pi \, i \, a\sqrt{k^2 + m^2 + n^2}} \quad ,$$

where $a \to +\infty$, $v << a^{2/3}$.

The best previous estimation of such sums was made by Vinogradov I.M. in [1]. The estimation is $S << a^{\frac{5}{3} + \varepsilon}$.

This inequality gives us the opportunity to estimate the error term $R(a)$ in the sphere problem as $R(a) << a^{\frac{4}{3} + \varepsilon}$.

We give the method that allows us to obtain the estimation

$$S << a^{\frac{5}{3} - \delta + \varepsilon} \quad , \quad \text{where} \, \delta = \frac{1}{45} \, .$$

This implies new error term $R(a)$ estimation in the sphere problem of the type

$$R(a) << a^{\frac{59}{45} + \varepsilon} \, .$$

References.
1. *Виноградов И.М.* Особые варианты метода тригонометрических сумм. - М.: Наука, 1976, 33стр.