THE BOUNDARY VALUE PROBLEM FOR QUASILINEAR PARABOLIC EQUATIONS WITH A LEVY LAPLACIAN FOR FUNCTIONS OF INFINITE NUMBER OF VARIABLES

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The Lévy Laplacian of $F(x)$ it the point x_0 is defined (if it exists) by the formula [1] $\Delta_L F(x_0) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}(F''(x_0)f_k, f_k)_H$, where function $F(x)$ defined on the Hilbert space H is twice strongly differentiable at a point x_0, $F''(x)$ is the Hessian of $F(x)$, and $\{f_k\}_1^\infty$ is an orthonormal basis in H.

Let $\overline{\Omega} = \Omega \cup \Gamma$ be a domain in H, $\Omega = \{x \in H : 0 \le Q(x) < R^2\}$, Γ is boundary and $\Gamma = \{x \in H : Q(x) = R^2\}$. The function $Q(x)$ is a twice strongly differentiable function such that $\Delta_L Q(x) = \gamma$, $\gamma > 0$ is positive constant. Consider the function $T(x) = \frac{R^2 - Q^2}{\gamma}$ possesses the following properties $0 < T(x)\frac{R^2}{\gamma}$, $\Delta_L T(x) = -1$ if $x \in \Omega$, and $T(x) = 0$ if $x \in \Gamma$. Let in a certain functional class $\mathcal{F}$ exists a solution of the boundary value problem for the heat equations $\frac{\partial V(t,x)}{\partial t} = \Delta_L V(t,x)$, $V(t,x) = G(t,x)$ on Γ, where $G(t,x)$ is a given function defined on H.

Consider the boundary value problem [2]

$$\frac{\partial U(t,x)}{\partial t} = \Delta_L U(t,x) + f_0(U(t,x)), \tag{1}$$

$$U(t,x) = G(t,x) \quad on\ \Gamma, \tag{2}$$

where $U(t,x)$ is a function on $[0,\mathcal{T}] \times H$, $f_0(\xi)$ is a given function of one variable and exist both primitive $\varphi(\xi) = \int \frac{d\xi}{f_0(\xi)}$ and its inverse function φ^{-1}.

Then solution of the boundary value problem (1), (2) is given by the equation

$$U(t,x) = \varphi^{-1}(T(x) + \varphi(V(t,x))),$$

where $V(t,x)$ is the solution of the boundary value problem for the heat equations.

References

1. Lévy P. *Problèmes concrets d'analyse fonctionnelle*. Paris: Gauthier–Villars, 1951.

2. Feller M. N. , Kovtun I. I. *Quasilinear parabolic equations with a Lèvy Laplacian for functions of infinite number of variables*, Methods of functional analysis and topology. Volume **14**. Number **2**, 2008, PP. 117-123.